Q-1) Find the minimum and maximum values of the function

$$f(x,y) = x^2 + y^2 - 2x + 2y + 5$$

on the closed disk

$$D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}.$$

Solution: We first find the critical values of the function.
$f_x = 2x - 2 = 0$ gives $x = 1$.
$f_y = 2y + 2 = 0$ gives $y = -1$.
This critical point $(1, -1)$ is in the domain D, so we calculate f there; $\boxed{f(1,-1) = 3.}$

Next we restrict f to the boundary of D which is parameterized as $x = 2\cos t$, $y = 2\sin t$, $t \in [0, 2\pi]$. We then have

$$\phi(t) = f(2\cos t, 2\sin t) = 4\sin t - 4\cos t + 9, \; t \in [0, 2\pi].$$

We find its critical points;
$\phi'(t) = 4\sin t + 4\cos t = 0$, $\tan t = -1$, $t = 3\pi/4$ or $7\pi/4$ both in $[0, 2\pi]$.

These give the values:
$\boxed{g(3\pi/4) = f(-\sqrt{2}, \sqrt{2}) = 9 + 4\sqrt{2} \approx 15}$.
$\boxed{g(7\pi/4) = f(\sqrt{2}, -\sqrt{2}) = 9 - 4\sqrt{2} \approx 3.3}$.

Checking the values in the boxes we find that the maximum value of f is $9 + 4\sqrt{2}$ and the minimum value is 3.