NAME: STUDENT NO:

Q-3) Change the order of integration of the following integral as indicated and also find the value of the integral by evaluating any of the integrals you find.
(Grading: each box=1 point, evaluation=7 points.)

$$\int_0^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{\sqrt{x^2+z^2}}^{3} z\,dy\,dx\,dz = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} z\,dy\,dz\,dx$$

$$= \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} z\,dz\,dx\,dy + \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} z\,dz\,dx\,dy.$$

Solution:

$$\int_0^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{\sqrt{x^2+z^2}}^{3} z\,dy\,dx\,dz = \int_{\boxed{-1}}^{\boxed{1}} \int_{\boxed{0}}^{\boxed{\sqrt{1-x^2}}} \int_{\boxed{\sqrt{x^2+z^2}}}^{\boxed{3}} z\,dy\,dz\,dx$$

$$= \int_{\boxed{0}}^{\boxed{1}} \int_{\boxed{-y}}^{\boxed{y}} \int_{\boxed{0}}^{\boxed{\sqrt{y^2-x^2}}} z\,dz\,dx\,dy +$$

$$\int_{\boxed{1}}^{\boxed{3}} \int_{\boxed{-1}}^{\boxed{1}} \int_{\boxed{0}}^{\boxed{\sqrt{1-x^2}}} z\,dz\,dx\,dy.$$

The last integral is easier to evaluate and we find $\frac{1}{6} + \frac{4}{3} = \frac{3}{2}$.