

NAME:

STUDENT NO:

Q-4) Let R be the region in the plane bounded by $y = x^2$, $y = x + 2$, and $x = 0$.
Let C be the boundary of R taken counterclockwise.

Let $F = (e^x + y^2 - \tan x + 1, \ln(y^3 + 1) + 2xy + x^3 - 7)$.

Calculate the work done by F along C , i.e. calculate $\int_C F \cdot T \, ds$.

Solution: Let $F = (M, N)$.

$$\begin{aligned} \int_C F \cdot T \, ds &= \int_C M \, dx + N \, dy \\ &= \iint_R (N_x - M_y) \, dA \quad (\text{Green's Theorem}) \\ &= 3 \iint_R x^2 \, dA \\ &= 3 \int_0^2 \int_{x^2}^{x+2} x^2 \, dy \, dx \\ &= \frac{44}{5}. \end{aligned}$$