NAME: STUDENT NO:

Q-5) Find the area of the region which lies in the plane $2x + 4y - z = 0$ and is bounded by the curve of intersection of this plane by the paraboloid $x^2 + y^2 - z = 9$.

Solution:

We want to calculate $d\sigma$ for the surface $F(x, y, z) = 2x + 4y - z = 0$.

The projection of the region on the xy-plane is given by $2x + 4y = x^2 + y^2 - 9$, which gives

$$(x - 1)^2 + (y - 2)^2 = 14.$$

Call this projection R.

$\nabla F = (2, 4, -1)$.
$|\nabla F| = \sqrt{4 + 16 + 1} = \sqrt{21}$.
$|\nabla F \cdot k| = 1$
Hence $d\sigma = \dfrac{|\nabla F|}{|\nabla F \cdot k|}\, dA = \sqrt{21}\, dA$.

Finally

$$\begin{aligned} \text{Area} &= \iint_R d\sigma \\ &= \sqrt{21} \iint_R dA \\ &= \sqrt{21} \cdot (\text{Area of } R) \\ &= \sqrt{21}\; 14\pi. \end{aligned}$$