Q-2) Assume that $3z + x + y^2 + xz^3 = 13$ defines z as a C^2 function of x and y around the point $(x, y, z) = (3, 2, 1)$. Find the values of z_x, z_y, z_{xy}, z_{yx}, z_{xx} and z_{yy} at the point $(x, y, z) = (3, 2, 1)$.

Solution:

Differentiate both sides of $3z + x + y^2 + xz^3 = 13$ implicitly with respect to x, taking y as the other independent variable and z as a differentiable function of x and y, to get

$$3z_x + 1 + z^3 + 3xz^2 z_x = 0, \tag{1}$$

which gives

$$z_x = -\frac{1 + z^3}{3(1 + xz^2)}. \tag{2}$$

Putting $(x, y, z) = (3, 2, 1)$ into the equation (**??**) or (**??**), we get

$$z_x = -\frac{1}{6}.$$

Similarly we get

$$z_y = -\frac{2y}{3(1 + xz^2)}, \tag{3}$$

and

$$z_y = -\frac{1}{3}.$$

Now using any of the equations (**??**),(**??**) or (**??**), differentiating implicitly and putting $(x, y, z, z_x, z_y) = (3, 2, 1,\ -1/6,\ -1/3)$, we get

$$z_{xx} = \frac{1}{24}, \quad z_{yy} = -\frac{1}{3}, \quad z_{xy} = z_{yx} = 0.$$