Q-3) Let S be the surface in $\mathbb{R}^3$ given by $f(x,y,z) = 0$ where $f(x,y,z) = 1+x^2+y^4-z$. Let $p_0 = (1/2, y_0, z_0)$ be a point on the surface such that the tangent plane to the surface S at p_0 passes through the origin. Find z_0.

Solution:

$$\nabla f(x,y,z) = (2x, 4y^3, -1).$$
$$\nabla f(p_0) = (1, 4y_0^3, -1).$$

Equation of the tangent plane to S at p_0 is

$$(1, 4y_0^3, -1) \cdot (x - \frac{1}{2}, y - y_0, z - z_0) = 0.$$

This passes through the origin so $(x,y,z) = (0,0,0)$ satisfies this equation giving

$$-\frac{1}{2} - 4y_0^4 + z_0 = 0. \tag{1}$$

Since $p_o = (1/2, y_0, z) =_0)$ is on the surface, we also have

$$\frac{5}{4} + y_0^4 - z_0 = 0. \tag{2}$$

Adding equations (**??**) and (**??**) we get

$$y_0^4 = \frac{1}{4}.$$

Putting this value into equation (**??**) or (**??**) we get

$$z_0 = \frac{3}{2}.$$