Q-1) For any $h \geq 0$ consider the region R_h in $\mathbb{R}^3$ bounded by the surfaces $z = (y+1)x^2$, $y = 0$, $y = 1$ and $z = h$. Find the volume of R_h.

Solution:

$$
\begin{aligned}
Volume &= 2 \int_0^1 \int_0^{\sqrt{h/(y+1)}} \int_{(y+1)x^2}^h dz\, dx\, dy \\
&= 2 \int_0^1 \int_0^{\sqrt{h/(y+1)}} h - (y+1)x^2\, dx\, dy \\
&= \frac{4h^{3/2}}{3} \int_0^1 \frac{dy}{\sqrt{y+1}} \\
&= \frac{8h^{3/2}}{3}(\sqrt{2} - 1) \\
&\approx (1.104)h\sqrt{h}.
\end{aligned}
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