Q-2) Let R be the region in $\mathbb{R}^3$ in the first octant bounded by the coordinate planes and the unit sphere. Evaluate the integral of the function $e^{(x^2+y^2+z^2)^{3/2}}$ on R.

Solution:

The problem requires that we pass to spherical coordinates.

$$\begin{aligned}
\iiint_R e^{(x^2+y^2+z^2)^{3/2}} dV &= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 e^{\rho^3}\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \\
&= \int_0^{\pi/2}\int_0^{\pi/2} \left(\frac{1}{3}\sin\phi \, e^{\rho^3}\Big|_{\rho=0}^{\rho=1} \right) d\phi \, d\theta \\
&= \frac{e-1}{3}\int_0^{\pi/2}\int_0^{\pi/2} \sin\phi \, d\phi \, d\theta \\
&= \frac{e-1}{3}\int_0^{\pi/2} \left(-\cos\phi\Big|_0^{\pi/2} \right) d\theta \\
&= \frac{e-1}{3}\int_0^{\pi/2} d\theta \\
&= \frac{(e-1)\pi}{6} \\
&\approx 0.899
\end{aligned}$$