Q-3) Consider the vector field $\vec{F} = \left(\frac{1}{x+y^2+z^3}, \frac{2y}{x+y^2+z^3} + 1, \frac{3z^2}{x+y^2+z^3} + 2z \right)$.
Calculate the work done by $\vec{F}$ along the path $C = C_1 + C_2 + C_3$.

C_1 is along the semicircle in the yz-plane with center at the origin and radius 2. C_1 follows this semicircle from $(0,-2,0)$ towards $(0,2,0)$ with $z \geq 0$.
C_2 goes from $(0,2,0)$ towards the point $(2,1,0)$ along the ellipse $\frac{3x^2}{16} + \frac{y^2}{4} = 1$ in the xy-plane.
C_3 goes from the point $(2,1,0)$ towards the point $(2,1,1)$ along a straight line.

Solution:

For the problem to be *reasonable*, $\vec{F}$ must be conservative! In fact we find that

$$\vec{F} = \nabla f, \quad \text{where} \quad f = \ln(x+y^2+z^3) + y + z^2,$$

and

$$\text{Work along } C = \int_C \vec{F} \cdot dr = f(2,1,1) - f(0,-2,0) = (\ln 4 + 2) - (\ln 4 - 2) = 4.$$