Q-4) Consider the curve of intersection of the surfaces $z = y$ and $z = x^2 + y^2$, and let C be the path on this curve from the origin to the point $(0, 1, 1)$ lying in the first octant. Calculate the work done by the vector $\vec{F} = (x, x^2, y + z)$ on the path C.

Solution:

A parametrization of the path C is $\vec{r}(t) = (\sqrt{t(1-t)}, t, t)$, $0 \le t \le 1$.

$$
\begin{aligned}
\text{Work along } C &= \int_C \vec{F} \cdot d\vec{r} \\
&= \int_0^1 \vec{F}(\vec{r}(t)) \cdot d\vec{r}(t) \\
&= \int_0^1 (\sqrt{t-t^2}, t - t^2, 2t) \cdot (\frac{1-2t}{2\sqrt{t-t^2}}, 1, 1)\ dt \\
&= \int_0^1 (\frac{1}{2} + 2t - t^2)\ dt \\
&= \frac{7}{6}.
\end{aligned}
$$