NAME: STUDENT NO:

Q-1) Show that $\lim_{n\to\infty} a_n = 0$, where $a_n = \dfrac{3\cdot 5\cdot 7\cdot \cdots \cdot (2n+1)}{4\cdot 9\cdot 14\cdots (5n-1)}$.

Solution:

We have two solutions to this problem. First the straightforward solution.

Observe that $\dfrac{2n+1}{5n-1} \le \dfrac{1}{2}$ for all $n \ge 3$. This gives

$$0 < a_n = \left(\frac{3\cdot 5}{4\cdot 9}\right)\left(\frac{7\cdots(2n+1)}{14\cdots(5n-1)}\right) \le \left(\frac{5}{12}\right)\left(\frac{1}{2}\right)^{n-2}.$$

By the Sandwich theorem, we have $\lim_{n\to\infty} a_n = 0$.

The other solution requires a little imagination. First observe that

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2n+3}{5n+4} = \frac{2}{5} < 1,$$

so $\sum_{n=1}^{\infty} a_n$ converges, forcing $\lim_{n\to\infty} a_n = 0$.