Q-2-b) Check the following two series for convergence: $\sum\limits_{n=1}^{\infty} \dfrac{n!}{n^n}$ and $\sum\limits_{n=1}^{\infty} \dfrac{n^n}{n!}$.

Solution:

For the first series

$$\frac{a_{n+1}}{a_n} = \frac{n^n}{(n+1)^n} = \frac{1}{(1+1/n)^n} \to \frac{1}{e} < 1 \quad \text{as} \ \ n \to \infty.$$

Therefore the first series converges by the ratio test. Hence the general term goes to zero as n goes to infinity. The general term of the second series is the reciprocal of the general term of the first series and hence goes to infinity as n goes to infinity. Then the second series diverges by the divergence test.