Q-3) Find the sum

$$\sum_{n=2}^{\infty} \frac{6}{(n-1)(n)(n+2)}$$

Solution:

$$\frac{6}{(n-1)(n)(n+2)} = \frac{-3}{n} + \frac{2}{n-1} + \frac{1}{n+2}.$$

Adding these from n = 2 to n = k we find

$$s_k = \sum_{n=2}^k \frac{6}{(n-1)(n)(n+2)} = \frac{7}{6} - \frac{4+3k}{k(k+1)(k+2)}.$$

Hence the sum is $\lim_{k\to\infty} s_k = \frac{7}{6}$.