NAME: STUDENT NO:

Q-4) Let r be the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{7n^2+1}{3n^3+n+81} x^n$.

a) Find r. *(10 points)*
b) Check the convergence of the series for $x = r$ and for $x = -r$. *(5+5 points)*

Solution:

Let $a_n(x) = \frac{7n^2+1}{3n^3+n+81} x^n$ and use ratio test for the absolute values. $\frac{|a_{n+1}(x)|}{|a_n(x)|} \to |x|$ as $n \to \infty$.

For absolute convergence we must have $|x| < 1$. So the radius of convergence is 1.

When $x = 1$, we have $\frac{|a_n(x)|}{1/n} \to 7/3$ as $n \to \infty$. Hence the series diverges at $x = 1$ by limit comparing with the Harmonic series.

When $x = -1$, we have an alternating series. The general term goes to zero and its absolute value decreases (see its first derivative below). Then the series converges at $x = -1$ by the alternating series test.

To see that the general term decreases to zero, let $f(t) = \frac{7t^2+1}{3t^3+t+81}$. Then $f'(t) = \frac{-21t^4 - 2t^2 + 1134t - 1}{(3t^3+t+81)^2}$ which is negative for all large t (in fact for all $t \geq 4$.)