

Q-2-b) Check the following two series for convergence: $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ and $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$.

Solution: First let $a_n = \frac{n^2}{e^n}$. We use the Ratio Test to check the convergence of $\sum_{n=1}^{\infty} a_n$.

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{1}{e} \cdot \left(1 + \frac{1}{n}\right)^2 = \frac{1}{e} < 1,$$

so this series converges. In particular the general term goes to zero as n goes to infinity.

The general term of the other series is the reciprocal of the general term of this series so its general term goes to infinity as n goes to infinity. Therefore $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ diverges.