Q-3) Find the sum
$$\sum_{n=3}^{\infty} \frac{1}{(n-2)\; n\; (n+3)}$$

Solution: First note that by using the partial fractions technique we can write
$$\frac{1}{(n-2)\; n\; (n+3)} = \frac{1/10}{n-2} + \frac{-1/6}{n} + \frac{1/15}{n+3}.$$

Letting $s_n = \displaystyle\sum_{k=3}^{n} \frac{1}{(k-2)\; k\; (k+3)}$ we calculate that
$$s_n = \frac{22}{225} + \frac{15(n+1)^3 + 5(n-1)^2 - 30n - 38}{30(n+3)(n+2)(n+2)(n)(n-1)}.$$

This then shows that
$$\sum_{n=3}^{\infty} \frac{1}{(n-2)\; n\; (n+3)} = \lim_{n\to\infty} s_n = \frac{22}{225}.$$