NAME: STUDENT NO:

Q-2 Consider the function $f(x, y) = x^3 + x^2 + y^2 - 4xy$ defined on $\mathbb{R}^2$. Find its critical points and classify the critical points using the second derivative test. Decide if the function has global maximum and minimum values.
(Grading: critical points=9 points, Second derivative test=9 points, global min/max=2 points.)

Solution: $f_x = 3x^2 + 2x - 4y = 0,\ f_y = 2y - 4x = 0.$
$f_y = 0$ gives $y = 2x$. Putting this into $f_x = 0$ gives $x = 0$ or $x = 2$.
Hence the critical points are $(0, 0)$ and $(2, 4)$.

$f_{xx} = 6x + 2,\ f_{yy} = 2,\ f_{xy} = -4.\ \nabla = f_{xx}f_{yy} - f_{xy}^2 = 12x - 12.$
$\nabla(0, 0) = 12 < 0$, so $(0, 0)$ is a saddle point.
$\nabla(2, 4) = 12 > 0$ and $f_{yy}(2, 4) = 2 > 0$, so $(2, 4)$ is a local minimum point.

Since the dominating term in f is x^3, the function is unbounded.