NAME: STUDENT NO:

Q-3) Let R be the region in $\mathbb{R}^3$ lying in the first octant, i.e. $x, y, z \geq 0$, and bounded by the cylindrical surfaces $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$. Write the limits of integration into the following boxes and evaluate the integral.
(Grading: each box=1 point, evaluation=8 points.)

$$\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} dz\,dy\,dx = \int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} dy\,dx\,dz.$$

Solution:

$$\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2}} dz\,dy\,dx$$

$$= \int_{0}^{1}\int_{0}^{\sqrt{1-z^2}}\int_{0}^{\sqrt{1-x^2}} dy\,dx\,dz.$$

The first integral is easier to evaluate:

$$\begin{aligned}
\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2}} dz\,dy\,dx &= \int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\,dy\,dx \\
&= \int_0^1 \left(y\sqrt{1-x^2}\Big|_0^{\sqrt{1-x^2}} \right) dx \\
&= \int_0^1 (1-x^2)\,dx \\
&= \left(x - \frac{x^3}{3}\Big|_0^1 \right) \\
&= \frac{2}{3}.
\end{aligned}$$