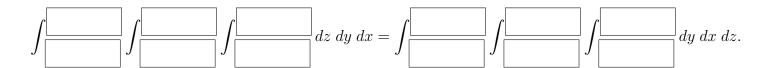
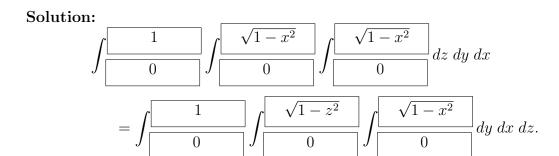
Q-3) Let R be the region in \mathbb{R}^3 lying in the first octant, i.e. $x, y, z \geq 0$, and bounded by the cylindrical surfaces $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$. Write the limits of integration into the following boxes and evaluate the integral. (Grading: each box=1 point, evaluation=8 points.)





The first integral is easier to evaluate:

$$\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{0}^{\sqrt{1-x^{2}}} dz \, dy \, dx = \int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \sqrt{1-x^{2}} \, dy \, dx$$

$$= \int_{0}^{1} \left(y\sqrt{1-x^{2}} \Big|_{0}^{\sqrt{1-x^{2}}} \right) \, dx$$

$$= \int_{0}^{1} (1-x^{2}) \, dx$$

$$= \left(x - \frac{x^{3}}{3} \Big|_{0}^{1} \right)$$

$$= \frac{2}{3}.$$