NAME: STUDENT NO:

Q-4) Assume that the equation $x^2+y^3+xy+x^2z+yz^2+z^5=11$ defines z as a C^∞ function of x and y around the point $(x,y,z)=(1,2,-1)$.

a) Write the equation of the tangent plane, at the point $(x,y,z)=(1,2,-1)$, to the surface defined by the above equation. *(10 points)*

b) Calculate the value of z_{xx} at the point $(x,y,z)=(1,2,-1)$. *(10 points)*

Solution:

Let $f(x,y,z)=x^2+y^3+xy+x^2z+yz^2+z^5-11$. $\nabla f(1,2,-1)=(2,14,2)$. The equation of the tangent plane at the given point is

$$(2,14,2)\cdot(x-1,x-2,x+1)=0,$$

which gives

$$x+7y+z=14.$$

For the second part, differentiating f with respect to x and treating z as a function of x and y gives

$$(2x+y+2xz)+(x^2+2yz+5z^4)\,z_x=0. \qquad (*)$$

Substituting $(x,y,z)=(1,2,-1)$ gives

$$z_x=-1.$$

Now differentiating both sides of $(*)$ again with respect to x we get

$$(2+2z+2xz_x)+(2x+2yz_x+20z^3z_x)\,z_x+(x^2+2yz+5z^4)\,z_{xx}=0$$

and substituting $(x,y,z)=(1,2,-1)$ and $z_x=-1$ we get

$$z_{xx}=10.$$