Q-5) Find the maximum and minimum values of $f(x, y, z) = x + \dfrac{y}{2} + \dfrac{z}{3}$ on the sphere $x^2 + y^2 + z^2 = (42)^2$.

Solution:

$\nabla f = (1, \dfrac{1}{2}, \dfrac{1}{3}) = \lambda(x, y, z)$ gives $x = \dfrac{1}{\lambda}$, $y = \dfrac{1}{2\lambda}$ and $z = \dfrac{1}{3\lambda}$.

Putting these into the equation of the sphere we get $\lambda^2 = \dfrac{1}{(36)^2}$. Hence $\lambda = \pm\dfrac{1}{36}$, and $(x, y, z) = \pm(36, 18, 12)$.

The maximum value is $f(36, 18, 12) = 49$ and the minimum value is $f(-36, -18, -12) = -49$.