NAME: STUDENT NO:

Q-2 Find the minimum and maximum values of $f(x, y) = x^2 + x + y^2 + y$ on the closed circular disk centered at the origin with radius $\sqrt{2}$.

Solution: Critical points are found by solving simultaneously $f_x = 2x + 1 = 0$ and $f_y = 2y + 1 = 0$. The only critical point is $(-1/2, -1/2)$ and is in the domain.

We parametrize the boundary by $x(\theta) = \sqrt{2}\cos\theta$ and $y(\theta) = \sqrt{2}\sin\theta$ where $0 \leq \theta \leq 2\pi$. Then we look for the critical points of

$$F(\theta) = f(x(\theta), y(\theta)) = 2 + \sqrt{2}(\cos\theta + \sin\theta),$$

where $\theta \in [0, 2\pi]$. Here $F'(\theta) = \sqrt{2}(-\cos\theta + \sin\theta) = 0$ for $\theta = \pi/4$ and $\theta = 5\pi/4$ in the range of θ. These give us the points $(1, 1)$ and $(-1, -1)$ on the boundary. We finally calculate the values of f at these critical values.

$f(-1/2, -1/2) = -1/2$,
$f(1, 1) = 4$,
f(-1,-1)=0.

We conclude that the minimum value of f is $-1/2$ and the maximum value is 4.