NAME: STUDENT NO:

Q-3) Change the order of integration of the following integral as indicated and also find the value of the integral by evaluating any of the integrals you find.
(Grading: each box=1 point, evaluation=7 points.)

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{2} x\,dz\,dy\,dx = \int_{\Box}^{\Box} \int_{\Box}^{\Box} \int_{\Box}^{\Box} x\,dz\,dx\,dy$$

$$= \int_{\Box}^{\Box} \int_{\Box}^{\Box} \int_{\Box}^{\Box} x\,dx\,dy\,dz + \int_{\Box}^{\Box} \int_{\Box}^{\Box} \int_{\Box}^{\Box} x\,dx\,dy\,dz.$$

Solution:

$$I = \int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{2} x\,dz\,dy\,dx = \int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{2} x\,dz\,dx\,dy$$

$$= \int_0^1 \int_{-z}^{z} \int_0^{\sqrt{z^2-y^2}} x\,dx\,dy\,dz + \int_1^2 \int_{-1}^{1} \int_0^{\sqrt{1-y^2}} x\,dx\,dy\,dz.$$

Either of the first two integrals can be converted to cylindrical coordinates to obtain

$$I = \int_{-\pi/2}^{\pi/2} \int_0^1 \int_r^2 r^2 \cos\theta\,dz\,dr\,d\theta = \frac{5}{6}.$$