NAME: STUDENT NO:

Q-4) Assume that the equation $x^3 + y^4 - xy^2 + 2xz - yz^3 - z^6 + 1 = 0$ defines z as a C^∞ function of x and y around the point $(x, y, z) = (-2, 1, -1)$.

a) Write the equation of the tangent plane, at the point $(x, y, z) = (-2, 1, -1)$, to the surface defined by the above equation. *(10 points)*

b) Calculate the value of z_x and z_{xx} at the point $(x, y, z) = (-2, 1, -1)$.
(Grading: 5+5 points. You can leave $z_{xx}(-2, 1, -1)$ unevaluated.)

Solution: Let $f(x, y, z) = x^3 + y^4 - xy^2 + 2xz - yz^3 - z^6 + 1$. Then $\nabla f(-2, 1, -1) = (9, 9, -1)$. The equation of the tangent plane to the surface $f(x, y, z) = 0$ at the point $(-2, 1, -1)$ is then

$$\nabla f(-2, 1, -1) \cdot (x + 2, y - 1, z + 1) = 0,$$

which gives

$$9x + 9y - z = -8.$$

As for the second part, differentiating both sides of $f(x, y, z) = 0$ with respect to x and treating z as a function of x and y gives

$$(3x^2 - y^2 + 2z) + (2x - 3yz^2 - 6z^5)\, z_x = 0. \qquad (*)$$

At the point $(-2, 1, -1)$ we get $z_x = -9$.

Finally differentiating both sides of $(*)$ with respect to x and treating z and z_x as functions of x and y gives

$$(6x + 2z_x) + (2 - 6yzz_x - 30z^4 z_x)\, z_x + ((2x - 3yz^2 - 6z^5)\, z_{xx} = 0.$$

Here putting $x = -2$, $y = 1$, $z = -1$ and $z_x = -9$ gives $z_{xx} = -1992$.