NAME: STUDENT NO:

Q-5) A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the Earth's atmosphere and its surface begins to heat. After one hour the temperature at the point (x, y, z) on the probe surface is

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600.$$

Find the hottest point on the probes's surface at that instant.

Solution: Let $g(x, y, z) = 4x^2 + y^2 + 4z^2 - 16$. We use Lagrange multipliers. The equation

$$\nabla T(x, y, z) = \lambda \nabla g(x, y, z)$$

gives $x = 0$ or $\lambda = 2$.

When $x = 0$ we can solve for λ and find that $\lambda = 0$ or $\lambda = \pm\sqrt{3}$.

In this case, when $x = 0$ and $\lambda = 0$ we find that $y = 4$ and $z = 0$.

When $x = 0$ and $\lambda = \sqrt{3}$ we find $y = -2$ and $z = -\sqrt{3}$.

When $x = 0$ and $\lambda = -\sqrt{3}$ we find $y = -2$ and $z = \sqrt{3}$.

In the other case, when $x \neq 0$ and $\lambda = 2$ we find that $x = \pm 4/3$ and $y = z = -4/3$.

Evaluating T at these critical points we find that:
$T(\pm 4/3, -4/3, -4/3) = 1928/4 \approx 642$. (max value)
$T(0, 4, 0) = 600$,
$T(0, -2, -\sqrt{3}) = 600 + 24\sqrt{3} \approx 641$
$T(0, -2, \sqrt{3}) = 600 - 24\sqrt{3} \approx 558$ (min value).