Date: June 20, 2013, Thursday NAME:.....

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Math 102 Summer 2013 – QUIZ # 3 – Section 001

Find a power series representation for $f(x) = \ln(5 - x)$ around $x = 0$ and determine the radius of convergence.

Solution:

Let $f(x) = \ln(5 - x)$. It is easy to prove by induction that $f^{(n)}(x) = -\dfrac{(n-1)!}{(5-x)^n}$ for $n \geq 1$.

Let $a_n = \dfrac{f^{(n)}(0)}{n!}\, x^n = \dfrac{1}{n5^n}\, x^n$. Using the ratio test we have

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{n}{n+1} \frac{1}{5} |x| = \frac{|x|}{5} < 1,$$

so the series converges absolutely for $|x| < 5$, making the radius of convergence to be 5.

Then we have

$$\ln(5 - x) = \ln 5 - \frac{1}{5}x - \frac{1}{50}x^2 - \frac{1}{375}x^3 - \frac{1}{2500}x^4 - \frac{1}{15625}x^5 - \cdots - \frac{1}{n5^n}\, x^n - \cdots,$$

for $|x| < 5$.