Date: July 4, 2013, Thursday NAME:.....

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Math 102 Summer 2013 – QUIZ # 7 – Section 001

Define

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ \\ 0 & \text{if } (x,y) = (0,0) \end{cases} .$$

(a) Show that $f_x(0,0)$ and $f_y(0,0)$ both exist.

Solution:

$$f_x(0,0) = \lim_{x\to 0} \frac{f(x,0) - f(0,0)}{x} = 0,$$
$$f_y(0,0) = \lim_{y\to 0} \frac{f(0,y) - f(0,0)}{y} = 0.$$

So these partial derivatives exist.

(b) Show that f is not differentiable at $(0,0)$.

Solution:

If f is differentiable at $(0,0)$, then it is continuous there but $\lim\limits_{\substack{x\to 0 \\ y=x}} f(x,y) = \lim\limits_{x\to 0} \dfrac{x^2}{x^2+x^2} = \dfrac{1}{2} \neq f(0,0)$.

So the function is not continuous at the origin and therefore not differentiable there.