Date: July 16, 2013, Tuesday NAME:.....

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Math 102 Summer 2013 – QUIZ # 10 – Section 001

Find the volume of the solid that lies above the paraboloid $z = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 2$.

Solution:
The two surfaces intersect along the plane $z = 1$. Therefore the shadow of the solid in the xy-plane is the unit circle $x^2 + y^2 = 1$. Then we can calculate the volume by passing to cylindrical coordinates.

$$\begin{aligned}
\iiint_{x^2+y^2\le 1} dV &= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} dz\,dy\,dx \\
&= \int_0^{2\pi}\int_0^1\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta \\
&= \int_0^{2\pi}\int_0^1 (\sqrt{2-r^2}-r^2) r\,dr\,d\theta \\
&= (2\pi)\left(-\frac{(2-r^2)^{3/2}}{3} - \frac{r^4}{4}\Big|_0^1\right) \\
&= \frac{4\sqrt{2}\pi}{3} - \frac{7\pi}{6} \\
&\approx 2.25
\end{aligned}$$