Date: July 18, 2013, Thursday NAME:..

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Math 102 Summer 2013 – QUIZ # 11 – Section 001

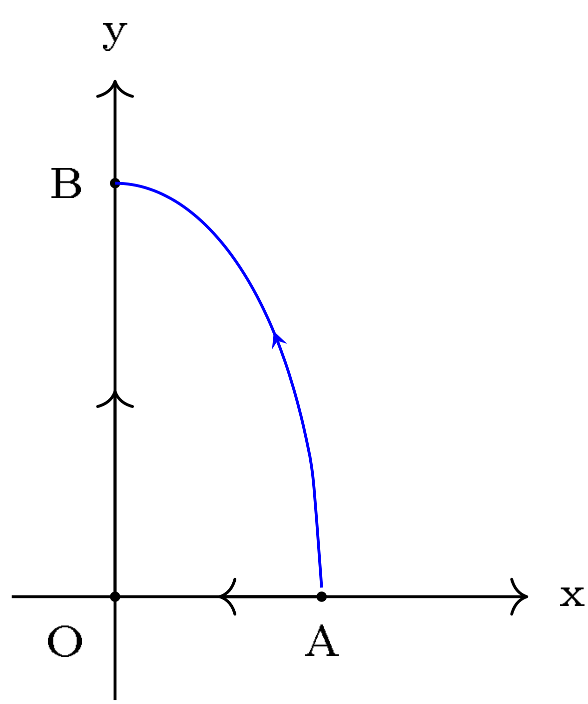
Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x,y) = (1+xy)e^{xy}\mathbf{i} + x^2e^{xy}\mathbf{j}$$

and C is the curve parametrized as follows:

$$\mathbf{r}(t) = \cos(t)\mathbf{i} + 2\sin(t)\mathbf{j},\ 0 \leq t \leq \frac{\pi}{2}.$$

Solution:



Let $\mathbf{F}(x,y) = (1+xy)e^{xy}\mathbf{i} + x^2e^{xy}\mathbf{j} = (P(x,y), Q(x,y))$.

Since $P_y = Q_x$ everywhere, the vector field $\mathbf{F}(x,y)$ is conservative on the plane. This means that the path integral of $\mathbf{F}(x,y)$ depends only on the end points. Since the given (blue) path starts at A and ends at B, we can choose any other path with these end points and the integral will have the same value.

We choose the path from A to O along the x-axis and continue along y-axis to B.

Let C_1 be parametrized by $\mathbf{r_1}(t) = (1-t, 0)$, $t \in [0,1]$, $d\mathbf{r_1} = (-dt, 0)$. (from A to O)
Let C_2 be parametrized by $\mathbf{r_2}(t) = (0, t)$, $t \in [0,2]$, $d\mathbf{r_2} = (0, dt)$. (from O to B)

Note that $\mathbf{F}$ evaluated along $\mathbf{r_1}(t)$ is $(1, (1-t)^2)$ and $\mathbf{F}$ evaluated along $\mathbf{r_2}(t)$ is $(1, 0)$.

Next we have

$$\begin{aligned}
\int_C \mathbf{F} \cdot d\mathbf{r} &= \int_{C_1} \mathbf{F} \cdot d\mathbf{r_1} + \int_{C_2} \mathbf{F} \cdot d\mathbf{r_2} \\
&= \int_0^1 (1, (1-t)^2) \cdot (-dt, 0) + \int_0^2 (1, 0) \cdot (0, dt) \\
&= \int_0^1 (-dt) + \int_0^2 0 \ dt \\
&= -1.
\end{aligned}$$