



Quiz # 10  
Math 102-003 Calculus

Date: April 30, 2014 Wednesday

STUDENT NAME:.....

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STUDENT NO:.....

**Q-1)** For each of the two series

$$(A): \sum_{n=1}^{\infty} (-1)^n (\sqrt[n]{2} - 1)^n, \text{ and } (B): \sum_{n=1}^{\infty} (-1)^n (\sqrt[n]{2} - 1),$$

answer the following questions:

Is the series convergent? If yes is it absolutely convergent?

*Show your work in detail. Only correct solutions will be graded; correct answers without justification are never graded.*

**Answer:**

**(A):** This series converges absolutely. Let  $disa_n = (-1)^n (\sqrt[n]{2} - 1)^n$ . Then

$$\lim_{n \rightarrow \infty} |a_n|^{1/n} = \lim_{n \rightarrow \infty} \sqrt[n]{2} - 1 = 0 < 1,$$

so the series with the absolute values converges by the Root Test. Hence the series is absolutely convergent.

**(B):** This series is convergent by the Alternating Series Test. However the series with the absolute values diverges by Limit Comparison Test with the harmonic series.

$$\lim_{n \rightarrow \infty} \frac{\sqrt[n]{2} - 1}{1/n} \stackrel{LH}{=} \lim_{n \rightarrow \infty} \frac{\sqrt[n]{2} \left( \frac{-\ln 2}{n^2} \right)}{-\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \sqrt[n]{2} \ln 2 = \ln 2 < \infty.$$