Quiz # 3
Math 102-003 Calculus

Date: February 26, 2014 Wednesday STUDENT NAME:.....

Instructor: Ali Sinan Sertöz STUDENT NO:.....

Q-1) In each of the following, decide if the limit exists or not and then prove your conclusion in detail. No grade will be assigned if you just say *converge* or *diverge*.

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y}{x^6+y^2}$

(b) $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{x^3yz^4}{x^6+y^2+z^4}$

(Grading: 5+5=10 points.)

Answer:

(a) Here put $y=\lambda x^3$ and use the Two Path Test to show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2} = \lim_{\substack{x\to 0\\ y=\lambda x^3}} \frac{\lambda x^6}{x^6+\lambda^2x^6} = \frac{\lambda}{1+\lambda^2}.$$

Since the limit depends on path, *the* limit does not exist.

(b) Here we can use the Sandwich Theorem.

$$0 \le \frac{|x^3yz^4|}{x^6+y^2+z^4} = \frac{z^4|}{x^6+y^2+z^4}|x^3y| \le |x^3y|.$$

In both cases we can use also my theorem:

Sertöz Theorem: *Let $a_1,\dots,a_N$ be non-negative integers, $m_1,\dots,m_N$ be positive integers and $c_1,\dots,c_N$ be positive real numbers, where $N>1$. Then*

$$\lim_{(x_1,\dots,x_N)\to(0,\dots,0)} \frac{x_1^{a_1}\cdots x_N^{a_N}}{c_1x_1^{2m_1}+\cdots+c_Nx_N^{2m_N}} \quad \textit{exists if and only if} \quad \sum_{i=1}^{N}\frac{a_i}{2m_i}>1.$$

Moreover, when the limit exists, then it is zero.

See http://sertoz.bilkent.edu.tr/depo/sertoz_theorem.htm