Quiz # 4
Math 102-003 Calculus

Date: March 5, 2014 Wednesday STUDENT NAME:.....

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Q-1) Let $F(x,y,z) = 3x^2 + xyz + y^3 - z^5$, and $\vec{v} = 2\vec{i} + 3\vec{j} - 6\vec{k}$.

(a) Find the directional derivative of F at the point $p = (-1,2,1)$ in the direction of the vector $\vec{v}$.

(b) Find a unit vector $\vec{u}$ such that the directional derivative of F in the direction of $\vec{u}$ at the point $p = (-1,2,1)$ is maximal.

(Grading: 5+5=10 points.)

Answer:

(a)

$$\begin{aligned}
\nabla F(x,y,z) &= (6x + yz, xz + 3y^2, xy - 5z^4) \\
\nabla F(-1,2,1) &= (-4, 11, -7) \\
|\vec{v}| &= \sqrt{4 + 9 + 36} = 7 \\
D_{\vec{v}} F(-1,2,1) &= \nabla F(-1,2,1) \cdot \frac{\vec{u}}{|\vec{u}|} \\
&= (-4,11,-7) \cdot \frac{1}{7}(2,3,-6) = \frac{19}{7}.
\end{aligned}$$

(b) The directional derivative of F at the point $p = (-1,2,1)$ is maximum in the direction of $\vec{w} = \nabla F(-1,2,1)$. Then $\vec{u} = \dfrac{\vec{w}}{|\vec{w}|} = \dfrac{1}{\sqrt{186}}(-4,11,-7)$.