



Quiz # 6
Math 102-003 Calculus

Date: March 12, 2014 Wednesday

STUDENT NAME:.....

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Q-1) Let $f(x, y) = e^{-xy}$, where x and y satisfy the condition $x^2 + 4y^2 = 8$.

- (a) Does this function have a maximum? If *yes* find it, if *no* explain why.
- (b) Does this function have a minimum? If *yes* find it, if *no* explain why.

(Grading: 5+5=10 points.)

Answer:

We first solve $x^2 + 4y^2 = 8$ for x to obtain $x = \pm 2\sqrt{2 - y^2}$. We then insert it into f to obtain two functions.

$$\phi(y) = \exp(2y\sqrt{2 - y^2}) \text{ and } \psi(y) = \exp(-2y\sqrt{2 - y^2}), \text{ where } -\sqrt{2} \leq y \leq \sqrt{2}.$$

We first treat $\phi(y)$.

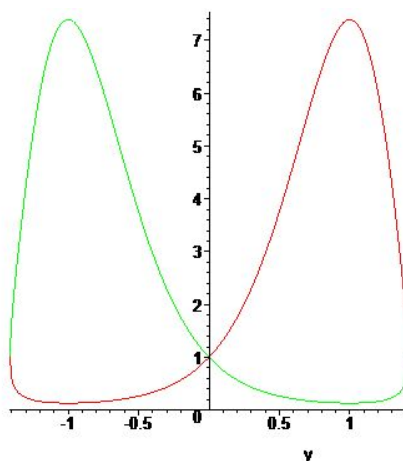
$$\phi'(y) = \phi(y) \left(-\frac{2y^2}{\sqrt{2 - y^2}} + 2\sqrt{2 - y^2} \right) = 0 \text{ gives } y = \pm 1.$$

We have

$$\phi(\pm\sqrt{2}) = 1, \quad \phi(1) = \frac{1}{e^2}, \quad \phi(-1) = e^2.$$

We get similar values for ψ .

Hence the minimum values of f is $f(2, 1) = f(-2, -1) = \frac{1}{e^2}$, and the maximum value is $f(2, -1) = f(-2, 1) = e^2$.



Here we have the graphs of ϕ and ψ .