Date: March 26, 2014 Wednesday

STUDENT NAME:..

Instructor: Ali Sinan Sertöz
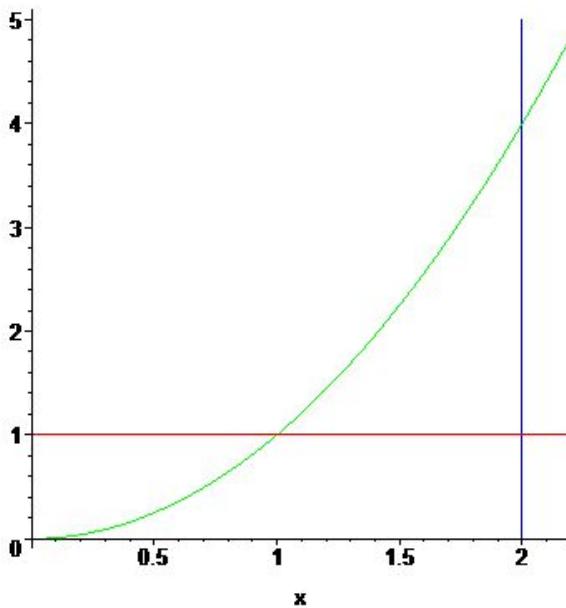
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Q-1) Evaluate the integral

$$\int_0^4 \int_{\sqrt{y}}^2 x^3 \cos(xy) \, dx \, dy.$$

Answer:

The graph shows $x = 1$, $y = x^2$ and $x = 2$. The region of interest is the region bounded by these three curves.

To evaluate this integral we must reverse the order of integration and for this we need the above figure.

$$
\begin{aligned}
\int_0^4 \int_{\sqrt{y}}^2 x^3 \cos(xy) \, dx \, dy &= \int_0^1 \int_0^{x^2} x^3 \cos xy \, dy \, dx + \int_1^2 \int_0^1 x^3 \cos xy \, dy \, dx \\
&= \int_0^1 \left(x^2 \sin xy \Big|_{y=0}^{y=x^2} \right) dx + \int_1^2 \left(x^2 \sin xy \Big|_{y=0}^{y=1} \right) dx \\
&= \int_0^1 x^2 \sin x^3 \, dx + \int_1^2 x^2 \sin x \, dx \\
&= \left(-\frac{1}{3} \cos x^3 \Big|_0^1 \right) + \left(-x^2 \cos x + 2 \cos x + 2x \sin x \Big|_1^2 \right) \\
&= -\frac{4}{3} \cos(1) + \frac{1}{3} - 2 \cos(2) + 4 \sin(2) - 2 \sin(1) \\
&\approx 2.39
\end{aligned}
$$