Quiz # 9
Math 102-003 Calculus

Date: April 16, 2014 Wednesday STUDENT NAME:..

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Q-1)

(a) Does the infinite sum $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ converge or diverge? Explain your answer.

(b) The infinite sum $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)(n+3)}$ converges. Find explicitly its sum.

Answer:

(a) We use Integral Test.

$$\int_1^{\infty} \frac{\ln x}{x^2}\,dx = \left(-\frac{\ln x}{x} - \frac{1}{x}\Big|_1^{\infty}\right) = 1.$$

Therefore the series converges.

Moreover we could also use the fact that $\ln x \leq x^{1/2}$ for $x \geq 1$ to show that

$$\frac{\ln n}{n^2} \leq \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}.$$

Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-test, our series converges by Direct Comparison Test.

(b) Using the partial fractions technique we first find that

$$\frac{1}{(n+1)(n+2)(n+3)} = -\frac{1}{2}\frac{1}{n+1} + \frac{2}{n+2} - \frac{3}{2}\frac{1}{n+3}.$$

Then a simple calculation shows that

$$\sum_{k=1}^{n} \frac{1}{(k+1)(k+2)(k+3)} = \frac{1}{4} - \frac{1}{2}\frac{3+2n}{(n+2)(n+3)}.$$

This finally shows that

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)(n+3)} = \frac{1}{4}.$$