Quiz # 2
Math 102-011 Calculus
February 20, 2015 Friday

Bilkent University

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Q-1)

(i) Apply the Alternating Series Test to the series $\sum_{n=0}^{\infty}(-1)^n \frac{n}{3^n}$. What do you conclude? *[50 points]*

(ii) Apply the Ratio Test to the series $\sum_{n=0}^{\infty}(-1)^n \frac{n^{2015}}{2015^n}$. What do you conclude? *[50 points]*

Show your work in detail. Correct answers without justification are never graded.

Answer:

(i) Let $a_n = \frac{n}{3^n}$. First we have

$$\lim_{n\to\infty}\frac{n}{3^n} = \lim_{x\to\infty}\frac{x}{3^x} \stackrel{LH}{=} \lim_{x\to\infty}\frac{1}{3^x \ln 3} = 0.$$

Next we show that a_n is decreasing. For all $n \geq 1$ we have

$$a_{n+1} = \frac{n+1}{3^{n+1}} = \frac{n}{3^n}\,\frac{n+1}{3n} = \frac{n}{3^n}\,\frac{n+1}{n+n+n} < \frac{n}{3^n} = a_n.$$

Hence by the Alternating Series Test this series converges.

(ii) Let $a_n = (-1)^n \frac{n^{2015}}{2015^n}$. Apply the ratio Test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{x\to\infty}\frac{(n+1)^{2015}}{2015^{n+1}}\,\frac{2015^n}{n^{2015}} = \lim_{x\to\infty}\frac{1}{2015}\left(1+\frac{1}{n}\right)^{2015} = \frac{1}{2015} < 1.$$

So the series converges absolutely.