Quiz # 3
Math 102-011 Calculus
February 27, 2015 Friday

Bilkent University

Instructor: Ali Sinan Sertöz NAME:

Q-1) In *each* of the following, find the radius of convergence of the series (*30 points*), and check for convergence at the end points (*10 points each*). Show your work in detail.

(i) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{x^n}{\ln n}$.

(ii) $\displaystyle\sum_{n=1}^{\infty} 2015^n\, n^{2015}(x-2015)^n$.

Answer:

(i) Let $a_n = (-1)^n\dfrac{x^n}{\ln n}$. Using the ratio test gives

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = |x|\cdot$$

For the radius of convergence, this must be less than one, so

$$|x| < 1 \text{ for convergence.}$$

Hence here the radius of convergence is 1. When $x = 1$, the series converges by the alternating series test. When $x = -1$, the series diverges by comparison with the harmonic series.

(ii) Let $a_n = 2015^n\, n^{2015}(x-2015)^n$. As above we have

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 2015\,|x-2015|.$$

For convergence this must be less than one. So we have

$$|x-2015| < \frac{1}{2015} \text{ for convergence.}$$

Hence the radius of convergence is $1/2015$. At the end points we have $|x-2015| = 1/2015$, so $a_n = \pm n^{2015}$ which does not go to zero as n goes to infinity. The series then diverges at both end points by divergence test.