

Quiz 3

Determine whether each of the following series converges or diverges.

a. $\sum_{n=1}^{\infty} \left(\frac{3}{2} - \sqrt[n]{n} \right)^n$

Set $a_n = \left(\frac{3}{2} - \sqrt[n]{n} \right)^n$.

Then $\lim_{n \rightarrow \infty} (a_n)^{1/n} = \frac{1}{2} < 1$, so the series converges by the n -th Root Test.

b. $\sum_{n=0}^{\infty} \frac{(n!)^3}{(3n)!}$

Set $a_n = \frac{(n!)^3}{(3n)!}$.

Then $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{1}{9} < 1$, so the series converges by the Ratio Test.

c. $\sum_{n=2}^{\infty} \frac{\sqrt{n^3 - 1}}{n^2}$

Set $a_n = \frac{\sqrt{n^3 - 1}}{n^2}$ and $b_n = \frac{1}{\sqrt{n}}$.

Then $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1$, and since $\sum_{n=2}^{\infty} b_n$ diverges (by p -test), our series diverges by the Limit Comparison Test.