Quiz # 5
Math 102-011 Calculus
27 March 2015, Friday

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YOUR NAME:

Q-1) Calculate the following limits. Show your work in detail. Correct answer without proper justification does not get any partial credits!

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y+xy^3}{x^4+6x^2y^2+y^4}$.

b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x^3y+xy^3)\ln(1+x^2+y^4)}{x^4+6x^2y^2+y^4}$.

: Grading is 50+50 points.

Answer:

a) Use the different path test. Let $y=\lambda x$. Then we have

$$\lim_{\substack{(x,y)\to(0,0)\\ y=\lambda x}} \frac{x^3y+xy^3}{x^4+6x^2y^2+y^4} = \lim_{x\to 0}\frac{\lambda+\lambda^4}{1+6\lambda^2+\lambda^4} = \frac{\lambda+\lambda^4}{1+6\lambda^2+\lambda^4},$$

and the limit depends on path. So *the* limit does not exist.

b) First note that

$$(x-y)^4\geq 0, \quad \text{which gives} \quad \frac{x^3y+xy^3}{x^4+6x^2y^2+y^4}\leq\frac{1}{4}.$$

Set

$$f(x,y)=\frac{x^3y+xy^3}{x^4+6x^2y^2+y^4}=\frac{xy(x^2+y^2)}{x^4+6x^2y^2+y^4}.$$

We see that

$$0\leq |f(x,y)| = f(|x|,|y|)\leq\frac{1}{4}.$$

From this we get

$$0\leq\left|\frac{(x^3y+xy^3)\ln(1+x^2+y^4)}{x^4+6x^2y^2+y^4}\right|\leq\frac{1}{4}\ \ln(1+x^2+y^4).$$

Since we have

$$\lim_{(x,y)\to(0,0)}\ln(1+x^2+y^4)=\ln 1=0,$$

we get by the sandwich theorem that

$$\lim_{(x,y)\to(0,0)} \frac{(x^3y+xy^3)\ln(1+x^2+y^4)}{x^4+6x^2y^2+y^4}=0.$$