Quiz # 6
Math 102-011 Calculus
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Bilkent University
Department of Mathematics

Instructor: Ali Sinan Sertöz

YOUR NAME:

In this quiz you can use fingers, calculators or smart phones to do your calculations.

Q-1) Consider the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Is f differentiable at $(0,0)$? If *not*, explain why. If *yes*, write the linearization $L(x,y)$ of f at $(0,0)$ and show that $|f(1,1/250) - L(1,1/250)| < 1/250$.

(b) Is f differentiable at $(3,4)$? If *not*, explain why. If *yes*, write the linearization $L(x,y)$ of f at $(3,4)$ and show that $|f(4,5) - L(4,5)| < 1/250$.

: Grading is 10+90 points.

: Hint: See grading!

Answer:

a) If f is differentiable at $(0,0)$, then it must be continuous there, but f is not continuous at $(0,0)$ since its limit as $(x,y) \to (0,0)$ does not exist by Sertöz Theorem: $\frac{1}{2} + \frac{1}{2} \not> 1$.

b) At $(3,4)$, the function f is given as a rational function whose denominator does not vanish. All its partial derivatives are also rational functions with denominator being the powers of the denominator of f. This means in particular that the first partial derivatives of f exist and are continuous at $(3,4)$, so f is differentiable there.

The linearization of f at $(3,4)$ is given by

$$L(x,y) = f(3,4) + f_x(3,4)(x-3) + f_y(3,4)(y-4).$$

We calculate and find that

$$f(3,4) = \frac{12}{25}, f_x(x,y) = \frac{y(y^2-x^2)}{(x^2+y^2)^2}, f(3,4) = \frac{28}{625}, f_y(x,y) = \frac{x(x^2-y^2)}{(x^2+y^2)^2}, f_y(3,4) = -\frac{21}{625}.$$

These give

$$L(x,y) = \frac{12}{25} + \frac{28}{625}(x-3) - \frac{21}{625}(y-4) = \frac{12}{25} + \frac{28}{625}\,x - \frac{21}{625}\,y.$$

Finally we observe that

$$|f(4,5) - L(4,5)| = \left|\frac{20}{41} - \frac{307}{625}\right| = \left|-\frac{87}{25625}\right| = 0.0033951... < 0.004 = \frac{1}{250}.$$