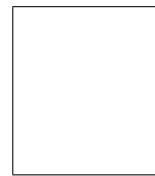




Quiz # 6
Math 102-011 Calculus
3 April 2015, Friday



Bilkent University
Department of Mathematics

Instructor: Ali Sinan Sertöz

YOUR NAME:

In this quiz you can use fingers, calculators or smart phones to do your calculations.

Q-1) Consider the function

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

- (a) Is f differentiable at $(0, 0)$? If *not*, explain why. If *yes*, write the linearization $L(x, y)$ of f at $(0, 0)$ and show that $|f(1, 1/250) - L(1, 1/250)| < 1/250$.
- (b) Is f differentiable at $(3, 4)$? If *not*, explain why. If *yes*, write the linearization $L(x, y)$ of f at $(3, 4)$ and show that $|f(4, 5) - L(4, 5)| < 1/250$.

: Grading is 10+90 points.

: Hint: See grading!

Answer:

a) If f is differentiable at $(0, 0)$, then it must be continuous there, but f is not continuous at $(0, 0)$ since its limit as $(x, y) \rightarrow (0, 0)$ does not exist by Sertöz Theorem: $\frac{1}{2} + \frac{1}{2} \not\rightarrow 1$.

b) At $(3, 4)$, the function f is given as a rational function whose denominator does not vanish. All its partial derivatives are also rational functions with denominator being the powers of the denominator of f . This means in particular that the first partial derivatives of f exist and are continuous at $(3, 4)$, so f is differentiable there.

The linearization of f at $(3, 4)$ is given by

$$L(x, y) = f(3, 4) + f_x(3, 4)(x - 3) + f_y(3, 4)(y - 4).$$

We calculate and find that

$$f(3, 4) = \frac{12}{25}, f_x(x, y) = \frac{y(y^2 - x^2)}{(x^2 + y^2)^2}, f_x(3, 4) = \frac{28}{625}, f_y(x, y) = \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}, f_y(3, 4) = -\frac{21}{625}.$$

These give

$$L(x, y) = \frac{12}{25} + \frac{28}{625}(x - 3) - \frac{21}{625}(y - 4) = \frac{12}{25} + \frac{28}{625}x - \frac{21}{625}y.$$

Finally we observe that

$$|f(4, 5) - L(4, 5)| = \left| \frac{20}{41} - \frac{307}{625} \right| = \left| -\frac{87}{25625} \right| = 0.0033951... < 0.004 = \frac{1}{250}.$$