Bilkent University
Department of Mathematics

Instructor: Ali Sinan Sertöz   YOUR NAME:

Q-1) Consider the differentiable functions

$$z = f(x, y), \quad \text{where} \quad x = g(s, t) \quad \text{and} \quad y = h(s, t).$$

Now consider the following data:

$f(0,0) = 19$	$f(3,5) = 14$	$f_x(0,0) = 23$	$f_x(3,5) = 16$	$f_y(0,0) = 29$	$f_y(3,5) = 18$
$g(0,0) = 3$	$g(3,5) = 2$	$g_s(0,0) = 7$	$g_s(3,5) = 6$	$g_t(0,0) = 11$	$g_t(3,5) = 8$
$h(0,0) = 5$	$h(3,5) = 4$	$h_s(0,0) = 13$	$h_s(3,5) = 10$	$h_t(0,0) = 17$	$h_t(3,5) = 12$

(i) Calculate $\dfrac{\partial z}{\partial s}(0,0)$.   **(ii)** Calculate $\dfrac{\partial z}{\partial t}(0,0)$.   **(iii)** Calculate $D_{\vec{u}}f(0,0)$, where $\vec{u} = \left(\frac{3}{5}, \frac{4}{5}\right)$.

: Grading is 35+35+30 points.

Answer:

(i)

$$
\begin{aligned}
\frac{\partial z}{\partial s}(0,0) &= f_x(3,5)g_s(0,0) + f_y(3,5)h_s(0,0) \\
&= 16 \cdot 7 + 18 \cdot 13 \\
&= 346.
\end{aligned}
$$

(ii)

$$
\begin{aligned}
\frac{\partial z}{\partial t}(0,0) &= f_x(3,5)g_t(0,0) + f_y(3,5)h_t(0,0) \\
&= 16 \cdot 11 + 18 \cdot 17 \\
&= 482.
\end{aligned}
$$

(iii)

$$
\begin{aligned}
D_{\vec{u}}f(0,0) &= f_x(0,0) \cdot \frac{3}{5} + f_y(0,0) \cdot \frac{4}{5} \\
&= 23 \cdot \frac{3}{5} + 29 \cdot \frac{4}{5} \\
&= 37.
\end{aligned}
$$