Quiz # 8
Math 102-011 Calculus
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YOUR NAME:

In this quiz you can use fingers, calculators or smart phones to do your calculations.
However show your work in detail. Correct answer without proper explanation does not receive any partial credits.

Q-1) Let $f(x,y) = x^3 + xy + y^3$. Answer the following questions about f. Remember that no correct answer without proper justification receives points.

(i) Does f have a global minimum/maximum? If *yes*, where?

(ii) Does f have local minimum/maximum? If *yes*, where?

(iii) Does f have saddle points? If *yes*, where?

: Grading is 20+40+40 points.

Answer:

(i) Along the x-axis, f takes the form $f(x,0) = x^3$ and is unbounded both from above and below. So f has neither a maximum nor a minimum.

(ii) For the critical points we have $f_x = 3x^2 + y = 0$ and $f_y = x + 3y^2 = 0$. Solving for y from the first equation and substituting into the second one we find that $x + 27x^4 = 0$. Thus we have $(0,0)$ and $(-1/3, -1/3)$ as the only critical points.

We have $f_{xx} = 6x$, $f_{yy} = 6y$, $f_{xy} = 1$ and $\Delta = 36xy - 1$.

We note that $\Delta(-1/3,-1/3) = 3 > 0$ and $f_{xx}(-1/3,-1/3) = -2 < 0$. Therefore $(-1/3,-1/3)$ is a local maximum for f.

(iii) We note that $\Delta(0,0) = -1 < 0$, so $(0,0)$ is a saddle point for f.

Here is a graph of f around the origin.