Quiz # 1
Math 102-001 Calculus
2 June 2016, Thursday
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Your Name: .....

Student ID: ..... Your Department: .....

Q-1) Use the integral test to check if the series $\sum_{n=3}^{\infty} \frac{\ln n}{n^2}$ converges or diverges.

Show your work in detail. Correct answers without justification are never graded.

Answer:

Let $f(x) = \frac{\ln x}{x^2}$ be defined for $x \geq 3$. Then $f'(x) = \frac{1 - 2\ln x}{x^3}$ and thus $f'(x) < 0$ for $x \geq 3$. Since we also have $f(x) > 0$ on this interval, we can apply the integral test.

$$\int_3^{\infty} \frac{\ln x}{x^2}\, dx = -\left(\frac{\ln x}{x}\Big|_3^{\infty}\right) + \int_3^{\infty} \frac{dx}{x^2} = \frac{\ln 3}{3} - \left(\frac{1}{x}\Big|_3^{\infty}\right) = \frac{\ln 3}{3} + \frac{1}{3} < \infty,$$

where we used integration by parts with $u = \ln x$ and $dv = \frac{dx}{x^2}$ for the first integral. Since the value of this integral is finite, the given series converges by the integral test.