



Quiz # 1  
Math 102-002 Calculus  
3 June 2016, Friday  
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Your Name: .....

Student ID: .....

Your Department: .....

**Q-1)** Use the integral test to check if the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$  converges or diverges.

*Show your work in detail. Correct answers without justification are never graded.*

**Answer:**

Let  $f(x) = \frac{1}{x(\ln x)^2}$  be defined for  $x \geq 2$ . Then  $f'(x) = -\frac{\ln x + 2}{x^2(\ln x)^3}$  and thus  $f'(x) < 0$  for  $x \geq 2$ . Since we also have  $f(x) > 0$  on this interval, we can apply the integral test.

$$\int_2^{\infty} \frac{1}{x(\ln x)^2} dx = \int_{\ln 2}^{\infty} \frac{du}{u^2} = -\left(\frac{1}{u}\right)\Big|_{\ln 2}^{\infty} = \frac{1}{\ln 2} < \infty,$$

where we used the substitution  $u = \ln x$  for the first integral. Since the value of this integral is finite, the given series converges by the integral test.