Quiz # 2
Math 102-**001** Calculus
9 June 2016, Thursday
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Solution Key

Bilkent University

Your Name: ....

Student ID: .... Your Department: ....

Q-1) For each of the following two series decide if it is "divergent", "conditionally convergent" or "absolutely convergent".

$$(\mathbf{a}) \quad \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}, \qquad\qquad (\mathbf{b}) \quad \sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^n}.$$

Show your work in detail. Correct answers without justification are never graded.

Answer:

(a) : Let $a_n = \dfrac{1}{(\ln n)^2}$. We first Limit Compare the series $\sum a_n$ with the divergent series $\sum 1/n$.

$$\lim_{n\to\infty} \frac{a_n}{1/n} = \lim_{n\to\infty} \frac{n}{(\ln n)^2} \stackrel{LH}{=} \lim_{n\to\infty} \frac{n}{2\ln n} = \infty.$$

This shows in particular that $a_n > 1/n$ for large n, hence the series $\sum a_n$ diverges.

On the other hand $a_n \downarrow 0$ as $n \to \infty$, so the series $\sum (-1)^n a_n$ converges. Finally, the given series is "conditionally convergent".

(b): Let $a_n = \dfrac{n!}{n^n}$. Use the Ratio Test.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{e} < 1.$$

Hence the given series is "absolutely convergent".