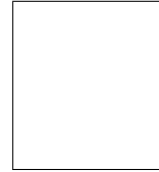




Quiz # 2  
 Math 102-001 Calculus  
 9 June 2016, Thursday  
 Instructor: Ali Sinan Sertöz  
**Solution Key**



Bilkent University

Your Name: .....

Student ID: .....

Your Department: .....

**Q-1)** For each of the following two series decide if it is “divergent”, “conditionally convergent” or “absolutely convergent”.

(a)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ ,

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^n}$ .

*Show your work in detail. Correct answers without justification are never graded.*

**Answer:**

**(a) :** Let  $a_n = \frac{1}{(\ln n)^2}$ . We first Limit Compare the series  $\sum a_n$  with the divergent series  $\sum 1/n$ .

$$\lim_{n \rightarrow \infty} \frac{a_n}{1/n} = \lim_{n \rightarrow \infty} \frac{n}{(\ln n)^2} \stackrel{LH}{=} \lim_{n \rightarrow \infty} \frac{n}{2 \ln n} = \infty.$$

This shows in particular that  $a_n > 1/n$  for large  $n$ , hence the series  $\sum a_n$  diverges.

On the other hand  $a_n \downarrow 0$  as  $n \rightarrow \infty$ , so the series  $\sum (-1)^n a_n$  converges. Finally, the given series is “conditionally convergent”.

**(b) :** Let  $a_n = \frac{n!}{n^n}$ . Use the Ratio Test.

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{n^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{1}{n}\right)^n} = \frac{1}{e} < 1.$$

Hence the given series is “absolutely convergent”.