Quiz # 3
Math 102-**001** Calculus
16 June 2016, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: .....

Student ID: ..... Your Department: .....

Q-1) Evaluate the sum $\displaystyle\sum_{n=0}^{\infty} \frac{n(n+1)}{3^n}$.

Show your work in detail. Correct answers without justification are never graded.

Answer:

We first observe that $\displaystyle\sum_{n=0}^{\infty} \frac{n(n+1)}{3^n} = \sum_{n=0}^{\infty} \frac{n^2}{3^n} + \sum_{n=0}^{\infty} \frac{n}{3^n}$.

Let $f(x) = \dfrac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots,\ |x| < 1.$

Then
$xf'(x) = x + 2x^2 + 3x^3 + \cdots + nx^n + \cdots,$
and
$x(xf'(x))' = x + 4x^2 + 9x^3 + \cdots + n^2x^n + \cdots.$

Finally $x(xf'(x))' + xf'(x) = \dfrac{2x}{(1-x)^3} = \dfrac{9}{4}$ when $x = 1/3$.