



Quiz # 3
Math 102-002 Calculus
17 June 2016, Friday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Your Name:

Student ID:

Your Department:

Q-1) Evaluate the sum $\sum_{n=0}^{\infty} \frac{n(n-1)}{4^n}$.

Show your work in detail. Correct answers without justification are never graded.

Answer:

We first observe that $\sum_{n=0}^{\infty} \frac{n(n-1)}{4^n} = \sum_{n=0}^{\infty} \frac{n^2}{4^n} - \sum_{n=0}^{\infty} \frac{n}{4^n}$.

Let $f(x) = \frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots$, $|x| < 1$.

Then

$$xf'(x) = x + 2x^2 + 3x^3 + \cdots + nx^n + \cdots,$$

and

$$x(xf'(x))' = x + 4x^2 + 9x^3 + \cdots + n^2x^n + \cdots.$$

Finally $x(xf'(x))' - xf'(x) = \frac{2x^2}{(1-x)^3} = \frac{8}{27}$ when $x = 1/4$.

Hence $\sum_{n=0}^{\infty} \frac{n(n-1)}{4^n} = \frac{8}{27}$.