Quiz # 4
Math 102-**001** Calculus
23 June 2016, Thursday
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Solution Key

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Your Name: .....

Student ID: ..... Your Department: .....

Q-1) Let $p = (1, 3, 3)$, $q = (3, 7, 10)$, and $r = (2, 2, 3 + \alpha)$, where α is a real number. Let π denote the plane passing through these three points.

(a) Write an equation for this plane in the form $Ax + By + Cz = D$, where A, B, C, D are constants (involving α).

(b) Find all values of α for which the plane π passes through the origin.

Show your work in detail. Correct answers without justification are never graded.

Answer:

We first note that the vectors $\vec{a} = q - p = (2, 4, 7)$ and $\vec{b} = r - p = (1, -1, \alpha)$ are parallel to the plane π. A normal to π is

$$\vec{s} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 7 \\ 1 & -1 & \alpha \end{vmatrix} = (4\alpha + 7, 7 - 2\alpha, -6).$$

Let $\vec{x} = (x, y, z)$. An equation for the plane is

$$\vec{s} \cdot (\vec{x} - p) = 0.$$

This gives

$$(4\alpha + 7)(x - 1) + (7 - 2\alpha)(y - 3) + (-6)(z - 3) = 0.$$

Simplifying this we get

$$(4\alpha + 7)x + (7 - 2\alpha)y - 6z = 10 - 2\alpha$$

for the required equation of the plane π. If the plane passes through the origin then $(x, y, z) = (0, 0, 0)$ satisfies this equation to give

$$0 = 10 - 2\alpha,$$

giving us

$$\alpha = 5.$$