Bilkent University

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Q-1) Let $p = (1, 2, 3)$, $q = (1 + \alpha, 10, 0)$, and $r = (1, 7, 4)$, where α is a real number. Let π denote the plane passing through these three points.

(a) Write an equation for this plane in the form $Ax + By + Cz = D$, where A, B, C, D are constants (involving α).

(b) Find all values of α for which the plane π passes through the origin.

Show your work in detail. Correct answers without justification are never graded.

Answer:

We first note that the vectors $\vec{a} = q - p = (\alpha, 8, -3)$ and $\vec{b} = r - p = (0, 5, 1)$ are parallel to the plane π. A normal to π is

$$\vec{s} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \alpha & 8 & -3 \\ 0 & 5 & 1 \end{vmatrix} = (23, -\alpha, 5\alpha).$$

Let $\vec{x} = (x, y, z)$. An equation for the plane is

$$\vec{s} \cdot (\vec{x} - p) = 0.$$

This gives

$$(23)(x - 1) + (-\alpha)(y - 2) + (5\alpha)(z - 3) = 0.$$

Simplifying this we get

$$23x - \alpha y + 5\alpha z = 23 + 13\alpha$$

for the required equation of the plane π. If the plane passes through the origin then $(x, y, z) = (0, 0, 0)$ satisfies this equation to give

$$0 = 23 + 13\alpha,$$

giving us

$$\alpha = -\frac{23}{13}.$$