Quiz # 5
Math 102-**001** Calculus
30 June 2016, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: .....

Student ID: ..... Your Department: .....

Q-1) Verify that $f(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of the Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$

Show your work in detail. Correct answers without justification are never graded.

Answer:

$$\frac{\partial f}{\partial x} = -\frac{x}{(x^2+y^2+z^2)^{3/2}}, \quad \frac{\partial^2 f}{\partial x^2} = \frac{2\,x^2 - y^2 - z^2}{(x^2+y^2+z^2)^{5/2}}.$$

By symmetry we can immediately write

$$\frac{\partial^2 f}{\partial y^2} = \frac{2\,y^2 - x^2 - z^2}{(x^2+y^2+z^2)^{5/2}} \quad \text{and} \quad \frac{\partial^2 f}{\partial z^2} = \frac{2\,z^2 - y^2 - x^2}{(x^2+y^2+z^2)^{5/2}}.$$

It now follows that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$