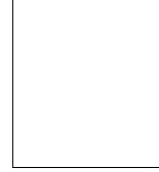




Quiz # 6  
Math 102-002 Calculus  
15 July 2016, Friday  
Instructor: Ali Sinan Sertöz  
**Solution Key**



Bilkent University

Your Name: .....

Student ID: .....

Your Department: .....

**Q-1)** Let  $f(x, y) = x^2 + 4xy + 5y^2 - 4x - 2y$ .

- (i) Find the critical points of  $f$ .
- (ii) Determine the nature of the critical points.
- (iii) Find the global minimum and maximum values of  $f$  if they exist.

*Show your work in detail. Correct answers without justification are never graded.*

**Answer:**

To find the critical points we must solve simultaneously the equations

$$f_x = 2x + 4y - 4 = 0 \quad \text{and} \quad f_y = 4x + 10y - 2 = 0.$$

Thus the only critical point is the sole solution to the above linear system which is  $(x, y) = (8, -3)$ .

To determine the nature of this critical point we need the second derivative test. For this we need

$$f_{xx} = 2, \quad f_{xy} = 4, \quad f_{yy} = 10, \quad \Delta = f_{xx}f_{yy} - f_{xy}^2 = 4.$$

Since  $\Delta > 0$ , this critical point is a local minimum.

We can write  $f(x, y) = (x + 2y)^2 + y^2 - 2(2x + y)$  and observe that as  $|x|$  and  $|y|$  increase, the values of the function go to  $+\infty$ . Hence there must be a global minimum which must occur at a critical point. Since we have only one critical point for this function, the global minimum whose existence we now know must occur there.

Hence  $f(8, -3) = -13$  is the global minimum value of this function.

Note that the existence of a single local minimum point does not guarantee that there is a global minimum. Check for example that the polynomial  $f(x, y) = x^2 + y^2(1 + x)^3$  has only one critical point at  $(0, 0)$  which is a local minimum but is not a global minimum. In fact  $\lim_{t \rightarrow \infty} f(-2, t) = -\infty$ .